Marker

Testing of Methods for the Determination of Young's Modulus by Bending

TESTING OF METHODS FOR THE DETERMINATION OF YOUNG'S MODULUS BY BENDING

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY ALBERT WASHINGTON MARKER

ENTITLED TESTING OF METHODS FOR THE DETERMINATION OF YOUNG'S MODULUS BY BENDING

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS IN PHYSICS

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T A B L E O F C O N T E N T S

I INTRODUCTION

Many attempts have been made to determine the exact relation between the stress and strain of a body. Among the early scientists this field of investigation was leading in thought and effort. As thought and reason did not have the dominant influence, men were handicapped in not having the benefit of a scientific past as men have today. Consequently, they proceeded on very far fetched assumptions. From common experience they noticed that a beam would bend under a stress but did not recognize a definite relation between the bending and the force producing it. This, however, was a minor part in this vast field. What took place within the bent object was a matter of serious thought and controversy. It took nearly a century and a half before men discovered and were able to calculate the exact relation between the longitudinal and lateral forces within the strained object. Theoretical and experimental investigations went hand in hand until at present there is a vast wealth of data showing the relation between stresses and the strains and also the phenomena which takes place within the distorted body. So many methods have been developed to show these relations that one has to choose those best adapted to his conditions.

It is the purpose of this investigation to determine what methods are best adapted for use in an advanced laboratory course to show the relation between stresses and strains.

II HISTORICAL

Many phases of science have made early and wonderful strides. Some of the thinkers of the middle ages were just as wonderful in their time as are our inventers and investigators of today. It is the fortune of the scientific world that men ventured the

persecutions of the time in the attempt to do something new. Indeed the dark age had shut out the light and even smothered intentionally the natural tendencies of men to think. Common everyday experience had the only chance in the scientific world to lead in discoveries and developments. The study of the bending of bars and beams or of anything took some primitive semi-organized form and was made a field of research under the name of elasticity. Galileo, in 1638, made some of the first steps in this direction. Knowing but little save that of common experience, he published short articles on this subject. He treated bodies as inelastic, cautiously proceeding with the attempt to establish some law of elasticity. His first experiments were with a beam, one end fastened to a wall, the stress being applied at the other. He concluded that the beam bent about an axis, in the plane of the wall, which was perpendicular to the plane of bending.

This was a great step in the beginning of an epoch of investigation. The spirit grew. Hooke of England continued the research and made a wonderful advancement. In 1660 he made definite advances in research and discoveries in elasticity which withstood the skeptical and critical transitional periods of progress. During all the time up to 1678 Hooke, vigilant and with increasing efforts, set forth the formulation of the renowned "proportionality law of strain and stress"- Ut tensio sic vis". To this day this law bears his name - unchanged and recognized universally. We may agree that Galileo gave the rudiments for its beginning while Hooke gave it the momentum which led to a definite form.

Unknown to Hooke in effort and investigation Mariotte of France, at the same time, was catching the spirit of the wave of a

coming age. He worked out a similar law. However, being a little late in publication, honor and title of the discoveries of the law must be given to Hooke. Yet Mariotte has earned for himself a place in the role of worthy recognition. He went a little further. Hooke made a mere enunciation, while Marriotte made the application of the law of elasticity.

This application has prime importance and is the first of the kind. Mariotte said that on bending, the bar was distorted as if it were made up of a number of layers - on one side of the flexure there was a contraction, on the other side an extension and in the middle a layer unaffected. This unaffected layer he located half way from the base of the beam and called it the neutral layer. This was exactly the problem Galileo tried to master. Mariotte assigned the axis of bending on the neutral filament at the point of support of the beam, perpendicular to its plane and one-half the distance from the base.

The next investigation of any importance, which was somewhat mathematical and theoretical, was made in 1705 by James Bernoulli a Swiss. He assumed that the force applied in bending was a resistance due to opposition offered from extension and contraction of the longitudinal filaments. Furthermore he assumed a definite relation to exist between the distorted axis and the curvature produced. The equation which he deduced expressed the couple as proportional to the curvature of the rod. On further investigation, knowing the flexure couple, James Bernoulli, Daniel Bernoulli and Euler worked out laws and differential equations which stated that the work done to bend a rod was proportional to the square of the curvature. These men added much by way of applying the mathematical and theoretical to the observed phenomena.

Later in the eighteenth century there was a number of men ready to carry the problem further. One of the important characters making noticeable progress and additions was a Frenchman by the name of Coulomb. In 1776 he published an article setting forth his deductions and conclusions. His work on the "elastica" of a rod was certainly new and far in advance of anything that had been done up to this time. Today we consider it as fundamental in every ramification of reasoning in the realm of elasticity. On experimentation he proved his equation of equilibrium, obtained by resolving the force implicated at a point on a normal cross section, into horizontal components. Doing this he was able to locate exactly the "neutral line" or the axis of equilibrium. Furthermore, he was able to calculate the moment of these component elastic forces. Former men had assumed the couple to be due to the resistance arising from extension and contraction of the longitudinal filaments. But he went further. From his equation and research he calculated and located exactly the magnitude of the different components of the resistance arising from the contraction and extension of the filaments. Not only in bending but also in torsional strain Coulomb advanced theories and proved that the torsional rigidity was proportional to the moment of inertia of the normal section about the axis of the fibre. And still further he was the first to make note of that kind of strain which we now call shear. He studied it however in a slightly different light than his predecessors. His consideration was made only in connection with rupture, that is, it "took place when the shear of the material is greater than a certain limit." "The shear was considered as a permanent set and not as elastic strain."

In harmony with the scientific spirit Thomas Young, an

Englishman, made in 1807, some important advances in the study of elasticity. He was the first to consider shear an elastic strain. This he called "detrusion." In making advances he enunciated that the resistance of a beam to shear and the resistance to extension and contraction were quite different. Some of his ideas pertaining to the relations of the strain to the stress are surprisingly approximate and even to this day stand in memoirs unshakeable by a critical age. Although he had expressed his ideas regarding these phenomena he failed to complete his mathematical conclusions as in the case of the modulus of rigidity for shear. He defined the modulus of elasticity of a substance as "a column of the same substance capable of producing a pressure on its base which is to the weight causing a certain degree of compression, as the length of the substance is to the diminution of its length." Today we define Young's modulus in terms of the weight of this column per unit of area of its base. This is a new method of the expression of thought. He ushered in a new epoch by clothing the theoretical with a physical garb and by introducing definition in the physical conception.

Considerable advancement in the field of elasticity has been due to the efforts of B. de Saint Venant, a Frenchman. He, however, majored in the field of torsion. He has cleared up a few points in bending which were of great service to his co-workers. During all this time the thought and old theories had taken some crude organized form. They were in the hypothetical stage. Saint Venant attempted to renovate them and took the offensive against two incomplete assumptions: (1) "that the strain consists of extension and contraction of longitudinal filaments; (2) that the stress consists of tension in the extended filaments and pressure along the

contracted filaments." Saint Venant took the premises for objection on the second assumption that on applying stress there must be a lateral contraction accompanying the longitudinal extension and also a lateral extension accompanying the longitudinal contraction. He restated several other old ideas giving them the experience gained by more research and organized information. Without doubt B. de Saint Venant has hastened all work along this line making way for later men to probe still deeper.

Looming up in many phases of Science Kirchhoff, a German, did no small amount in the field of elasticity. In 1859 he published a treatise which was somewhat exceptional in character. From the experience of all his predecessors he began with differential equations. He deducted formulae which were expressions of the energy of bending in terms of extension and contraction and the components of curvature. He verified his formulae by using thin bars and continued experimenting showing that he was equally right in case of thicker material.

Kirchhoff's method of attack and his theories aroused considerable discussion among his contemporaries. Clebsch, B. de Saint Venant, Kelvin, Tait, J. Boussinesq and others suggested substitutes and different methods of reasoning in the Kirchhoff controversy. Clebsch's modification of Kirchhoff's equation and experiments for the flexural couple of the curvature of the "central line" was verified by later investigators and has been permanently established.

It is needless to state more historically as to the energy and efforts expended in the development of this all important topic. Who of these men is "primus inter pares" is not easily

determined. Everyone is influential. Other men like Poisson, Navier, Cauchy, Euler, Lagrange and Lord Rayleigh should have worthy mention in the development of this immense field of elasticity.

III APPARATUS

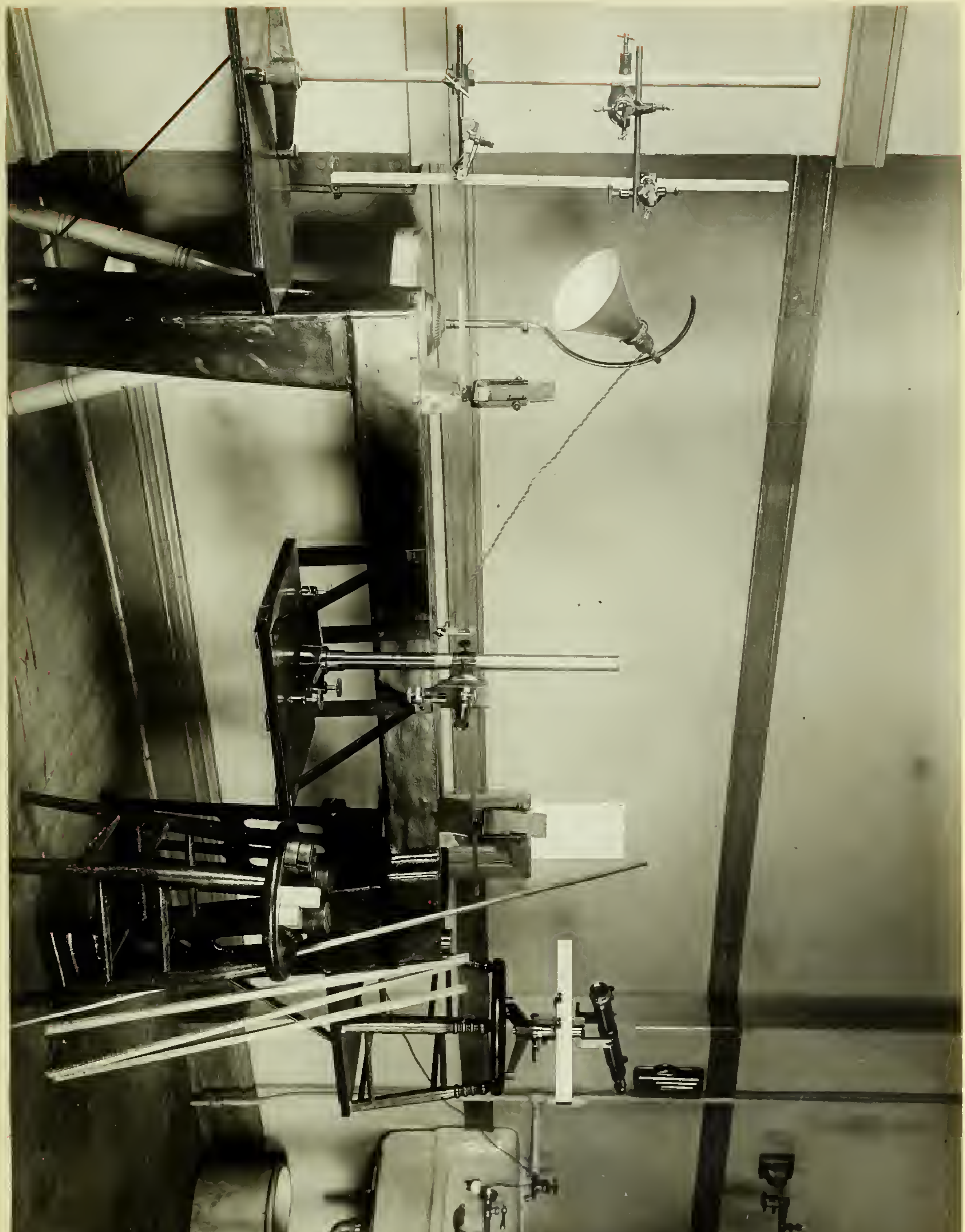
The accompanying pictures show the nature of the apparatus in detail. The base upon which all the essential parts are supported is a lathe bed. On each end is a shelf rigidly fastened on which are placed a scale and microscope. At the side near the middle is a third shelf firmly fastened to support a microscope. The bar in question is placed on moveable knife edges which are firmly bolted below the base of the top by means of thumb screws. The pans for the applied weight for bending may be suspended anywhere on a knife edge.

The mirrors used were supported by a frame fastened to the bar and so fixed that they could be rotated about an axis at right angles to the bar. The silvered side of these mirrors was used for the reflecting surface. By means of these the vertical scale is reflected and read through the telescope. A beam of light was directed upon the scale to make a distinct image. The microscope near the middle of the bar was direct reading to a thousandth of a centimeter and could be estimated to a ten-thousandth of a centimeter.

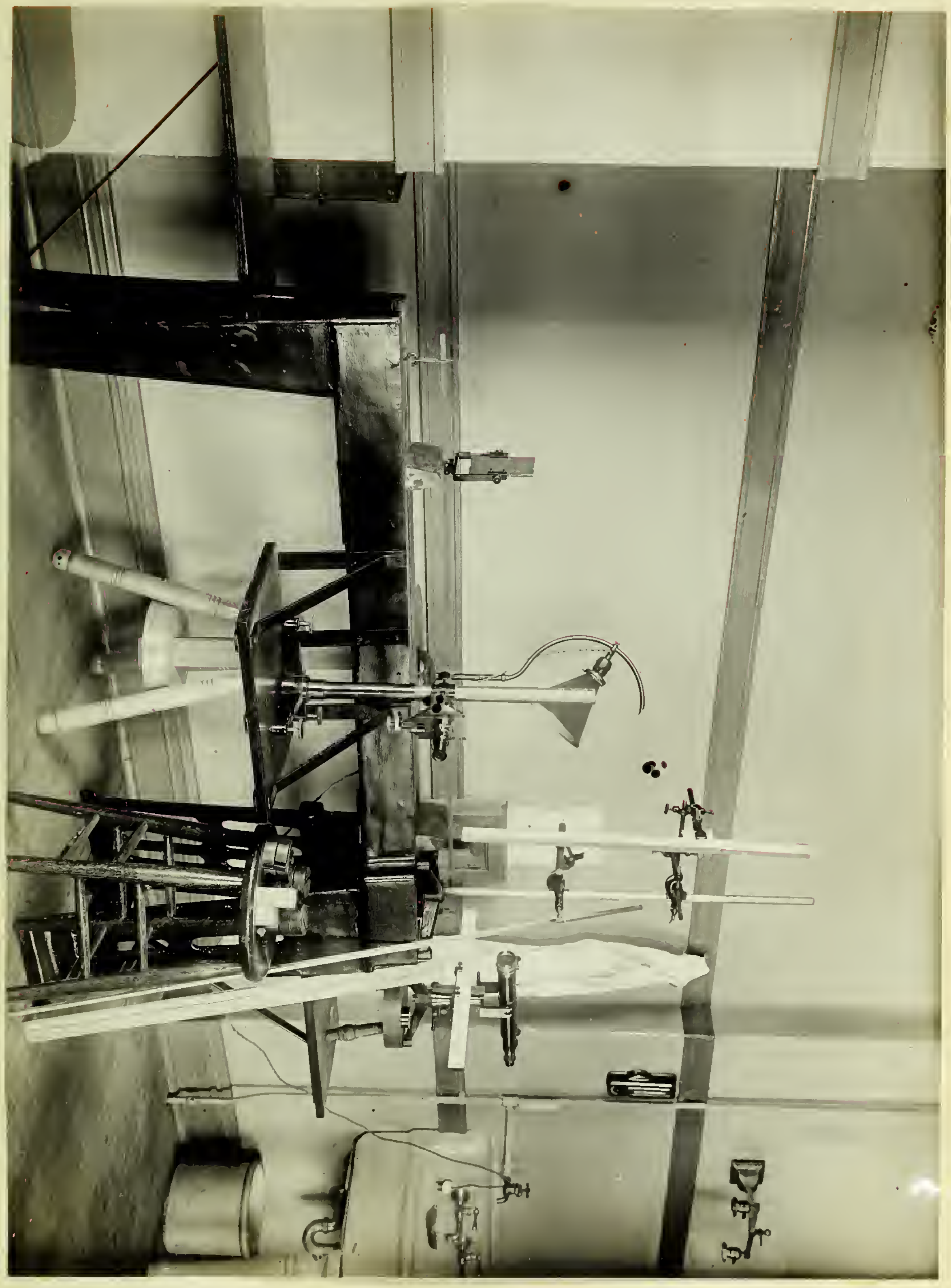
IV METHODS

As stated in the introduction, the object of this investigation is to study methods of determining Young's Modulus of elasticity for the purpose of being able to select those methods best adapted to laboratory use. To this end the following methods, all of which are standard, have been tested.

I. Load applied in middle of rod supported at both ends, depression



Apparatus arranged for Methods I and II



Apparatus arranged for Methods III and IV

of middle point measured.

II. Load applied in middle of rod supported at both ends, deflection of beam of light from mirrors directly over knife edge measured.

III. Load applied at ends, outside of knife edges, deflection of depression measured by mirror over one knife edge and scale over the other.

IV. Load applied at ends, outside of knife edges, depression measured at middle point.

Method I - Theory

In the first method considered the stress is applied at the center, between the knife edges. The displacement is observed by means of a microscope provided with a vernier. The following drawing shows the nature and principle of the apparatus.

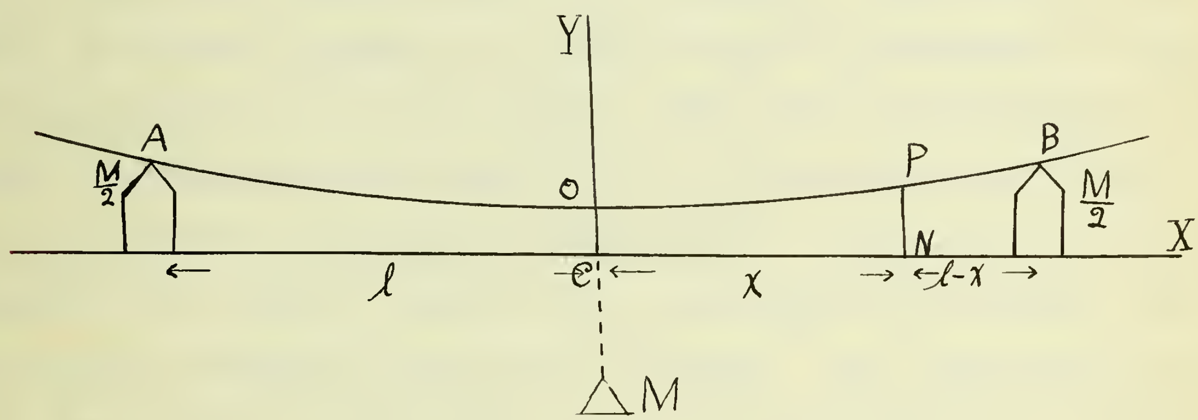


Fig. 1

Let AB (Fig.1) be the rod bent by applying stress M at C. Let P be any point on the rod, and its coordinates with respect to the axes CX and CY through the center C be x and y. Let AB = 2l. In case any weight is applied at M each knife edge in addition to the weight of the beam pushes up with a force of 1/2 M. Hence if G be the bending moment at P about the point B,

$$G = 1/2M(1-x)\text{dynes-cm.} \tag{1}$$

If a tension of T dynes per square cm. is applied to each end of a uniform bar the stress on any plane perpendicular to the axis is a uniform tension of T dynes per square cm. The increase in length due to the pull is in proportion to the whole length of the rod and also to the stress applied. The ratio of the longitudinal stress to the elongation is called Young's Modulus and is denoted by E. That is

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\text{elongation}} = \frac{T}{e} \tag{2}$$

$$E = \frac{T}{e} \tag{3}$$

and

$$T = Ee. \tag{4}$$

Now let ABC, Fig. 2, be a cross section of a beam with RH in its plane and bending about it as its axis. There is one filament, the neutral filament, in the bar which is unchanged in length. Let the plane cut it at any point O. Then from the rectangular axes OX and OY, parallel and perpendicular to RH draw MP and NP, the x and y coordinates from any strained point P. Produce PM and NO cutting HH' and K and R. Then MK is ρ, the radius, the distance from the neutral filament to the axis of bending. The longitudinal filament through OM is unstretched but on passing outward the filaments are stretched in proportion to the distance from OM. Consequently S_1, the length of any stretched portion through P, is to S its original length, as PK is to MK. That is

$$e = \frac{S_1 - S}{S} = \frac{(\rho + y) - \rho}{\rho} = \frac{y}{\rho} \tag{5}$$

or

$$e = \frac{y}{\rho} \tag{6}$$

From (4) and (6)

$$T = Ee = E\frac{y}{\rho} \tag{7}$$

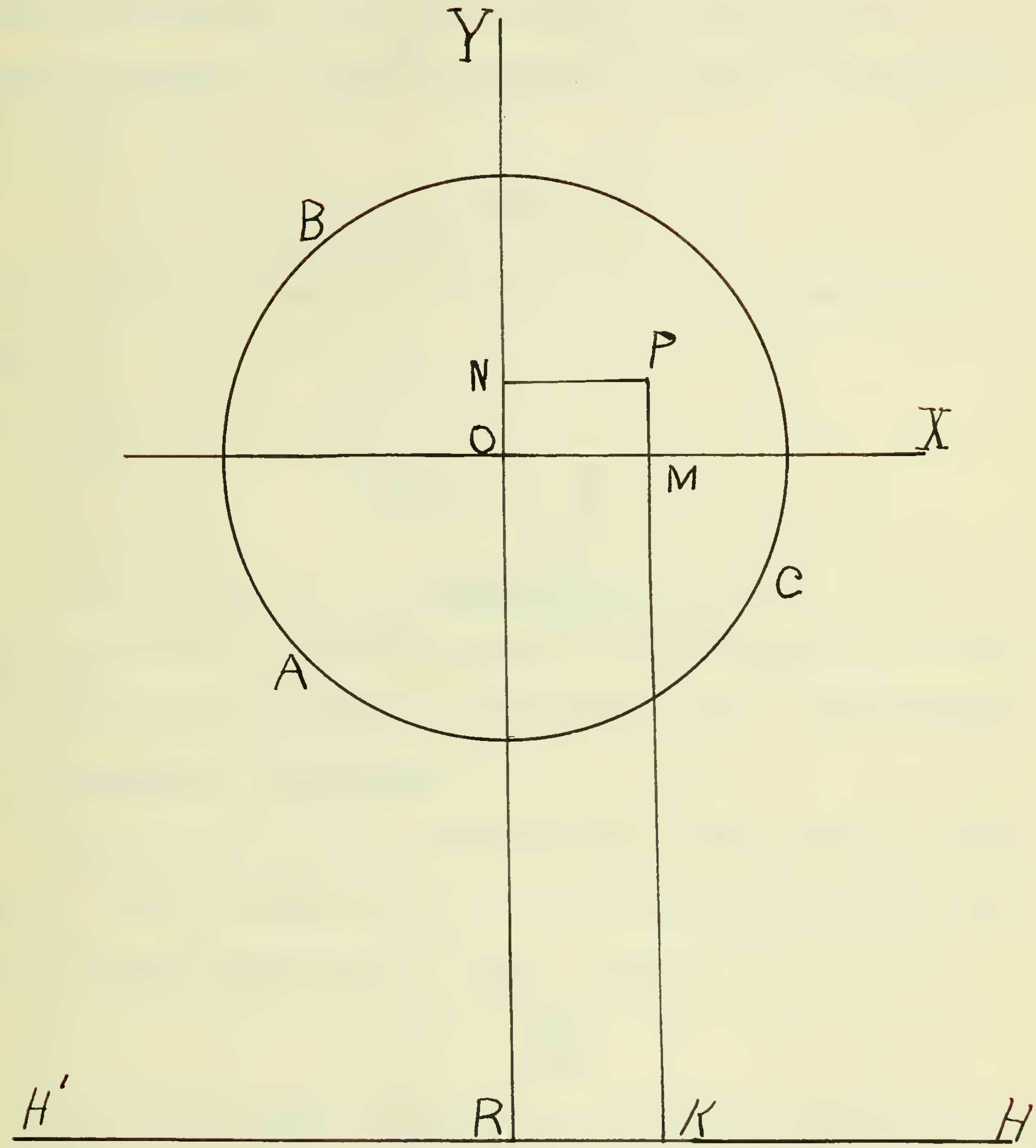


Fig. 2

If N is the total force acting at this cross section plane and if α is the area of a filament

$$N = \Sigma T \alpha \ = E \Sigma \alpha \frac{y}{\rho} \tag{8}$$

The sum of the moments about the OX axes, Fig.2, when the longitudinal filaments slip with respect to each other is equal to the "bending moment". Then the moment of force G about OX at any distance y is

$$G = \Sigma T \alpha y = \frac{E \Sigma \alpha \ y^2}{\rho} = \frac{EI}{\rho} \tag{9}$$

where $I = \Sigma \alpha y^2$, the moment of inertia of the cross section about the axes OX.

Then from (1) and (9)

$$1/2M(1-x) = \frac{EI}{\rho} \tag{10}$$

The value of ρ must first be determined. If the bending is slight, the form of the rod may be represented by a single differential equation in which the curvature, $1/\rho$, is expressed in terms of Cartesian coordinates.

Let x and y be the coordinates of any point P (Fig.3) on the curve. Let ρ be the radius of curvature, and ψ the angle the tangent at P makes with the OX axes. Then

$$\tan \psi \ = \frac{dy}{dx} \tag{11}$$

Take some other point O, an increment distance ds and dψ from P. Then $\qquad \frac{dx}{dy} = \cos \psi ,$

and $\qquad d\psi = \frac{ds}{\rho}$

$$\frac{d\psi}{ds} = \frac{ds}{ds\rho} \quad = \quad \frac{1}{\rho}$$

Then

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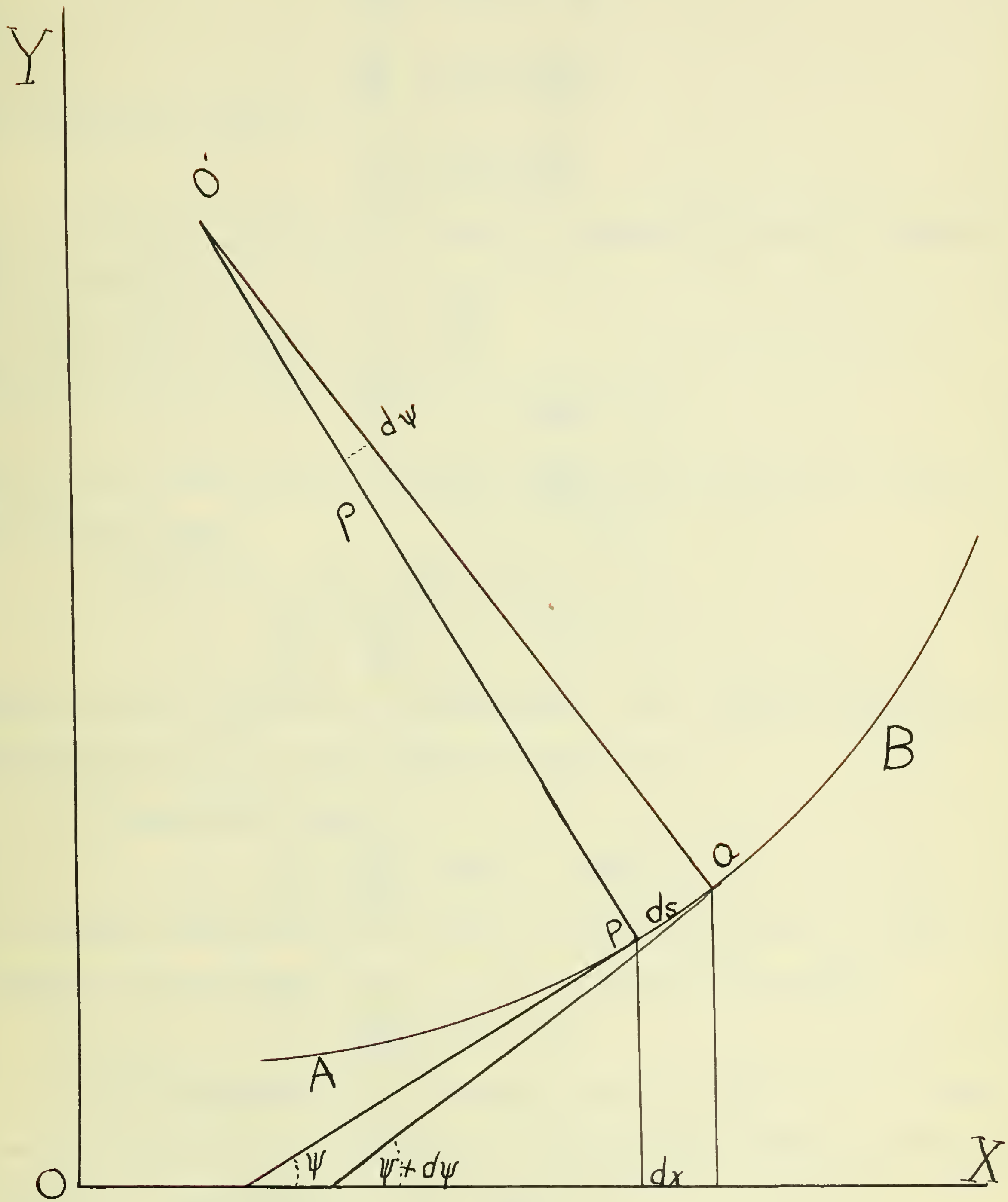

Fig. 3

$$\frac{1}{\rho} = \frac{d\psi}{ds} = \frac{d\psi}{dx} \cdot \frac{dx}{ds} = \cos\psi \frac{d\psi}{dx} \tag{12}$$

Differentiating (11)

$$\frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{dx} \tag{13}$$

and

$$\frac{d\psi}{dx} = \cos^2\psi \frac{d^2y}{dx^2}$$

Substituting in (12)

$$\frac{1}{\rho} = \cos^3 \frac{d^2y}{dx^2} \tag{14}$$

If ψ is small $\cos^3\psi$ may be replaced by unity. Consequently (14) becomes

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} \tag{15}$$

Then from (15) and (10)

$$\frac{d^2y}{dx^2} = \frac{1}{\rho} = \frac{Mg}{2EI}(1-x) \tag{16}$$

This is the differential equation from which the form of the rod may be determined.

Integrating (16) with respect to X

$$\frac{dy}{dx} = \frac{Mg}{2EI}(1x - 1/2x^2) + K \tag{17}$$

where K is the constant of integration. At C (Fig.1) the rod is horizontal and x = 0, hence $\frac{dy}{dx} = 0$ and K = 0.

Integrating again

$$y = \frac{Mg}{2EI}(1/21x^2 - 1/6x^3) + N \tag{18}$$

N is another constant whose value is zero, for when x = 0 the point P moves to the OX axes, i.e., y = 0. Hence (18) becomes

$$y = \frac{Mg}{12EI}(31x^2 - x^3) \tag{19}$$

This expression is true only for CB, (Fig.1). If point P moves to the left of OY axes X is negative and $1/2M_g(1-x)$ becomes $1/2M_g(1+x)$. Consequently (19) changes to:

$$y = \frac{Mg}{12EI}(31x^2 + x^3) \tag{20}$$

(19) and (20) are numerically equal but only opposite in signs.

If the rod (Fig. 1) is bent the depression, y, at C is equal to the elevation, h, at B above the OX axes. But if considered here, at the knife edge, X = l

Consequently from (19)

$$h = \frac{Mgl^3}{6EI} \qquad (21)$$

and

$$E = \frac{Mgl^3}{6hI} \text{ dynes per cm.}^2 \qquad (22)$$

For a rectangular rod of width 2a cm. and depth 2b cm., the vertical side, the moment of inertia I is

$$I = \frac{4}{3} ab^3 \text{ cm}^4 \qquad (23)$$

Therefore

$$E = \frac{Mgl^3}{8ab^3h} \text{ dynes per sq.cm.} \qquad (24)$$

Method II - Theory

In this method a mirror, mounted in a suitable frame to rotate about a horizontal axis perpendicular to the bending beam, is clamped over each knife edge. The scale is on a shelf firmly fastened at the end of the lathe bed while the telescope is on a similar shelf at the other end. The scale is reflected by the two mirrors

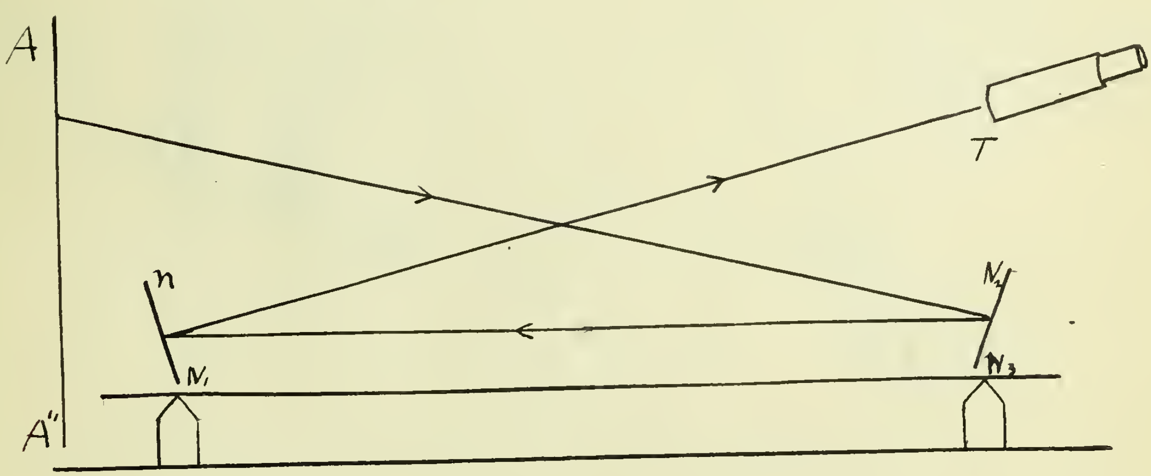

Fig. 4

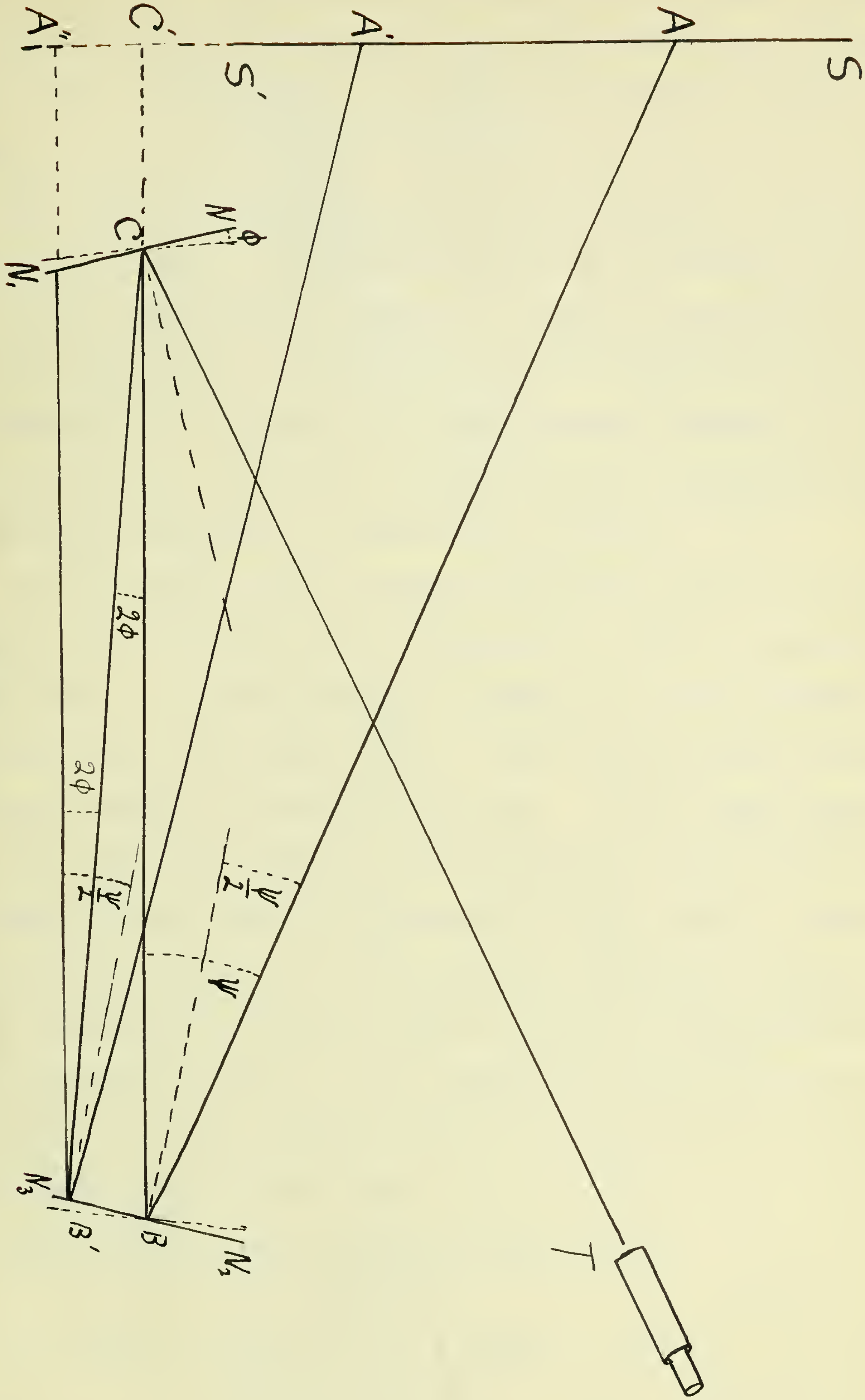

Fig. 5.

as is shown in Fig.4. The mirrors are turned at an angle in order to direct the reflected light in the right direction and to obtain sufficient scale reading space on account of the mirror next to the telescope.

When applying a mass M, Fig.4, at the center, the rod is bent into a curved form. If an inclination, ϕ is produced the displacement at the center may be stated in terms of the tangent which is not easily obtained. If, however, x be the reading of the scale observed through the telescope and D the distance, between mirror N_2N_3 and scale, Fig.5, and a the distance between the mirrors NN_1 and N_2N_3, ϕ may be expressed in terms of x, D and a.

Let the light, Fig.5, passing from the scale at A to B be reflected to C and then to the telescope T. First suppose that mirror NN_1 alone is rotated through an angle ϕ in the clockwise direction and the light coming from T. The reflected ray CB would pass along CB' making the angle B'CB of 2ϕ. If the angle ABC or TCB be ψ, the angle of incidence from T is $\frac{\psi}{2}$ and after the rotation of mirror the incident angle of ray CB' at B' is $\frac{\psi}{2} - 2\phi$. If, however, mirror N_2N_3 be rotated in the anti-clockwise direction through an angle ϕ, $\frac{\psi}{2} - 2\phi$ would be decreased by ϕ, that is, the incident angle becomes $\frac{\psi}{2} - 3\phi$ Then if B'A' is the reflected ray the angle CB'A' is $\psi - 6\phi$ If A"B' be drawn parallel to BC' the angle CB'A" is equal to 2ϕ. Hence angle A"B'A' is $\psi - 4\phi$

Then if the scale is perpendicular to C'B

$$AC' = D \tan \psi \tag{25}$$

$$A'A'' = D \tan(\psi - 4\phi) \tag{26}$$

$$C'A'' = A \tan 2\phi. \tag{27}$$

Then x the observed displacement through telescope is

$$X = AA' = D \tan\psi - [D \tan(\psi - 4\phi)] + a \tan 2\phi$$

If the deflection is very small the angle is equal to the tangent, that is,

$$X = 4D\phi + 2a\phi \tag{28}$$

or

$$\phi = \frac{x}{4D+2a} \tag{29}$$

Now from (17)

$$\frac{dy}{dx} = \frac{Mg}{2EI}(1x - 1/2x^2) + K \tag{30}$$

where K is zero as shown above.

From Fig.3, $\frac{dy}{dx}$ = tangent of any angle made with the OX axes. That is according to (4) and (6)

$$\frac{dy}{dx} = \phi = \frac{Mg}{2EI}(1x - 1/2x^2) \tag{31}$$

If the depression at the middle point C is h cm., it is the same as the height at the knife edge B (Fig.1). Let the point P move to B, then 1 = x and equation (7) becomes

$$\phi = \frac{Mgl^2}{4EI} \tag{32}$$

Solving for E

$$E = \frac{Mgl^2}{4I\phi} \tag{33}$$

Substituting in (33) the value ϕ from (29)

$$E = \frac{Mgl^2}{2IX}(2D+a) \tag{34}$$

If we wish to use the entire distance between the knife edges instead of one half of it, 1 = 21 and (34) becomes:

$$E = \frac{4\ Mgl^2}{2IX}(2D+a) \tag{35}$$

If the rod has a rectangular cross-section of width 2b and vertical depth 2d, the moment of inertia, I is

$$I = \frac{4}{3}d^3b \text{ cm}^4 \tag{36}$$

Therefore by substituting in (35) the value I of (36)

$$E = \frac{3Mgl^2}{2d^3b}(2D+a) \text{ dynes per cm.}^2 \tag{37}$$

Method III - Theory

In this method the "bending moment" is the same at any point on the bar, thus producing uniform distortion. To produce this condition the stresses are applied outside of the knife edges and equally distant from the same. A microscope just opposite the center C is placed on the middle shelf to observe the displacement.

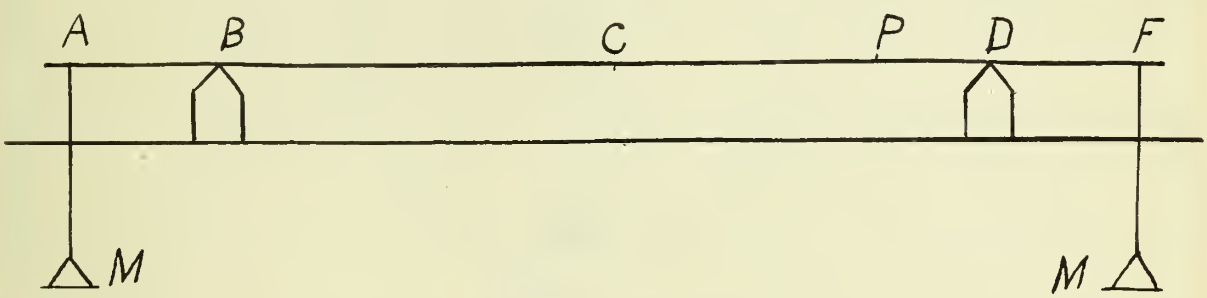

Fig. 6

Let the mass M be the force applied at A and F, Fig.6, a distance p from the knife edges B and D. Now let P be any point between B and D, and if G be the bending moment at this point P due to the weight applied,

$$G = M(PF - PD)g = Mpg \text{ dynes - cm.} \tag{38}$$

In this equation the weight of the bar is not taken into consideration. If the bar bends due to its own weight it is very slight and will not change the constant value of G at any point of the bar. Consequently it may be neglected.

Now if the "moment of inertia" of the transverse section of the rod about an axis at right angles to the plane of bending and through the center of gravity, be I cm^4 and as shown in equation (9)

$$E = \frac{G\rho}{I} \tag{39}$$

where ρ is the radius of curvature, we have from (38) and (39)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$E = \frac{Mgp\rho}{I} \text{ dynes per cm.}^2 \tag{40}$$

The value of ρ may be expressed in terms of the displacement h as observed at the center by the microscope. Let RV = 1

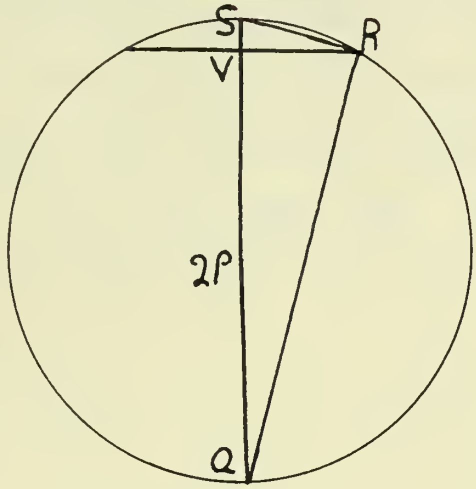

Fig. 7

(Fig.7) be one half the length of the rod, SQ = 2ρ where ρ is the radius of curvature, and SV equal the displacement h due to the stress applied, then by geometry

$$1^2 = h(2\rho - h). \tag{41}$$

But h of $(2\rho - h)$ being very small as compared with 2ρ, may be neglected. Consequently

$$\rho = \frac{1^2}{2h} \tag{42}$$

Then from (40) and (42)

$$E = \frac{Mgp1^2}{2hI} \text{ dynes per cm.}^2 \tag{43}$$

If the rod is of rectangular cross section with the horizontal side, 2a cm. and vertical 2b cm. the "moment of inertia" about an axis perpendicular to the cross section through the center of gravity be I cm^4,

$$I = \frac{4}{3} ab^3 \tag{44}$$

Therefore substituting in (43) the value of I from (44)

$$E = \frac{3}{8} \cdot \frac{M_g p l^2}{ab^3 h} \text{ dynes per cm.}^2 \qquad (45)$$

Method IV - Theory

In this method the experiment is the same as in Method III except that the displacement is found by means of a mirror, the plane of which is placed over one knife edge (Fig.8) and scale, the face of which is over the other knife edge. The reflection is observed by a telescope placed back of the scale on a shelf firmly fastened to the lathe bed.

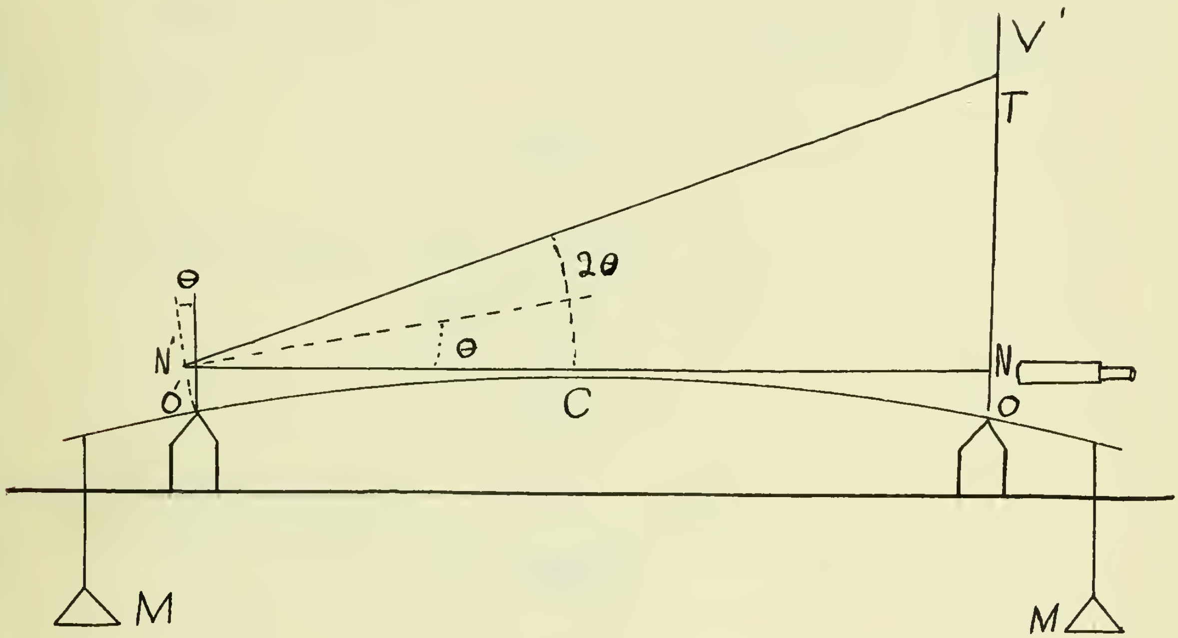

Fig. 8

It is obvious that the tangent to the bar at the middle point c (Fig.8) remains always parallel to OO'. Then if the tangent at O' is deflected through an angle θ when a load M is applied at each pan, ρ being the radius of the curved rod, and l one-half of the distance between the knife edges, the angle θ may be expressed in terms of ρ and l as;

$$\theta = \frac{l}{\rho} \qquad (46)$$

If the tangent at O' is deflected through an angle θ when a mass M is placed on each pan, then N'T is turned through an angle 2θ. If z be the displaced distance as read on the vertical scale NV', angle TN'N, 2θ may be expressed as

$$2\theta = \frac{z}{2l} \tag{47}$$

Now since the angle as observed is twice that through which the mirror is rotated

$$\theta = \frac{z}{4l} \tag{48}$$

Substituting this value of θ in equation (46) and solving for ρ, we have

$$\rho = \frac{4l^2}{z} \tag{49}$$

Then equating (42) and (49)

$$\frac{4l^2}{z} = \frac{l^2}{2h}$$

or

$$h = \frac{z}{8} \tag{50}$$

Substituting this value in (45)

$$E = \frac{3Mpgl^2}{ab^3z} \tag{51}$$

V DATA

The data in Tables I, II and III was obtained by testing the samples according to Methods I and II, while that in Tables IV, V and VI was obtained by Methods III and IV. In the determination of Young's Modulus only one sample of each of two substances, steel and brass, was used. These bars were carefully selected in order to secure samples of uniform dimension and without any distortion. They were 150 cm. in length and about one square centimeter in cross-section.

Each line of Tables II and III represent a series of readings similar to that shown in Table I. The first line of Table II gives only the essential terms for the formula as observed and given in Table I. In like manner Table IV is a series of readings for one trial determination of Young's Modulus and is the same as in the fourth line of Table V. The other determinations were obtained in a similar manner.

The observations are entered in Tables I, II, and III according to the following notation.

Each side of cross-section of the steel bar = 2a = 2b = .96 cm.

Each side of cross-section of the brass bar = 2a = 2b = .95 cm.

Weight applied = W gms.

Direct reading for applying weight = d cm.

Direct reading for removing weight = d_1 cm.

Difference of scale reading due to 500 gms. = T cm.

Mean of the difference of scale readings = Q cm.

Mean of the difference for microscope = Q' cm.

Young's Modulus = E dynes per square cm.

Distance between knife edges = K cm.

Distance between mirrors = a cm.

Distance between the scale and mirror N_2N_3 (Fig.5) = D cm.

The direct reading d and d_1 of the microscope are not recorded.

TABLE I

Steel bar

K = 66.3 cm. a = 66.8 cm. D = 130.1 cm.

	Method II					Method I		
W	d	T	d_1	T	Q	T'	T'	Q'
0	34.40		34.40					
500	35.00	.60	35.01	.61	.605	.022	.021	.0215
1000	35.62	.62	35.64	.63	.625	.020	.022	.0210
1500	36.23	.61	36.26	.62	.615	.021	.022	.0215
2000	36.87	.64	36.90	.64	.640	.020	.021	.0200
2500	37.49	.62	37.50	.60	.610	.022	.020	.0210
3000	38.10	.61	38.10	.60	.605	.020	.023	.0215
				Av.	.6166		Av.	.02108

TABLE II

Steel bar

Method II					Method I	
Q	K	A	D	$E \times 10^{-12}$	Q'	$E \times 10^{-12}$
.6160 cm	66.3 cm	66.8 cm	130.10 cm	1.9779	.02108 cm	1.9952
1.960	106.3	105.85	149.85	1.9804	.08775	1.9761
1.166	86.21	87.2	139.90	1.9820	.0466	1.9829
1.1316	86.28	77.2	134.00	1.9642	.0464	1.9949
			Av.	1.9760		1.9871

TABLE III

Brass Bar

Method II					Method I	
Q	K	A	D	$E \times 10^{-12}$	Q'	$E \times 10^{-12}$
2.338 cm	86.50 cm	85.90 cm	143.00 cm	1.0740	.0929 cm	1.0445
2.977	96.50	95.85	140.10	1.0566	.1292	1.0469
2.148	82.45	81.95	140.90	1.0434	.0798	1.0560
1.793	78.40	78.10	131.00	1.0520	.0686	1.0564
4.038	108.80	108.40	146.10	1.0621	.1864	1.0391
3.637	104.70	104.00	144.10	1.0663	.1654	1.0436
3.301	100.80	100.00	142.00	1.0675	.1470	1.0526
3.138	98.70	98.10	141.00	1.0646	.1380	1.0481
3.802	106.80	105.90	144.80	1.0707	.1746	1.0495
			Av.	1.0619		1.0485

In Tables IV, V and VI, the observations are entered according to the following notation.

Each side of cross-section of steel bar = 2a = 2b = .96 cm

Each side of cross-section of brass bar = 2a = 2b = .95 cm

Weight applied = W gms.

Direct scale reading for applying weight = d cm.

Direct scale reading for removing weight = d_1 cm.

Difference of scale reading due to 500 gms. = T cm.

Mean of the difference of scale readings = Q cm.

Mean of the difference for microscope = Q' cm.

Young's Modulus = E dynes per sq.cm.

Distance from knife edge to point of application of stress = p cm.

Distance between knife edges = K cm.

The direct scale readings, d and d_1, of the microscope are not recorded.

TABLE IV

Steel Bar

K = 68.1 cm. p = 37 cm.

Method IV							Method III	
W	d	T	d_1	T	Q	T'	T'	Q'
0	67.40		67.40					
500	66.81	.59	66.80	.60	.595	.074	.075	.0745
1000	66.22	.59	66.20	.60	.595	.074	.075	.0745
1500	65.62	.59	65.60	.60	.595	.074	.075	.0745
2000	65.02	.60	65.00	.60	.600	.075	.074	.0745
2500	64.44	.58	64.44	.56	.570	.075	.072	.0735
				Av.	.591		Av.	.0743

TABLE V

Steel bar

Method IV				Method III	
Q	K	p	$E \times 10^{-12}$	Q'	$E \times 10^{-12}$
.704 cm	94.18 cm	23.9 cm	2.0841	.0937 cm	1.9578
.7033	84.31	29.0	2.0286	.08991	1.9335
.705	84.00	29.2	2.0372	.0897	2.0010
.591	68.10	37.0	2.0100	.0743	1.9953
.496	58.08	42.2	1.9870	.0619	1.9901
.440	54.05	44.0	2.0270	.0564	1.9723
.554	64.05	40.0	2.0501	.0710	2.0181
.6061	74.00	34.0	2.1256	.08075	1.9953
.624	74.00	34.0	2.0651	.0809	1.9816
		Av.	2.0483	Av.	1.9894

TABLE VI

Brass bar

Method IV				Method III	
Q	K	p	$E \times 10^{-12}$	Q'	$E \times 10^{-12}$
1.240 cm.	74.00 cm	34 cm	1.0814	.1554 cm	1.0810
1.354	84.12	29	1.1014	.1729	1.0710
1.427	90.12	26	1.0654	.1806	1.0522
1.426	96.50	23	1.0842	.1805	1.0710
1.387	86.50	28	1.0895	.1752	1.0183
		Av.	1.0848	Av.	1.0707

VI SOURCES OF ERROR

In order to get consistent results the following cautions should be carefully observed.

General

1. The knife edges supporting the stress tend to travel on adding and removing weights, thus changing the distance between the knife edge and point of application of stress. This may be greatly remedied by observing special care when changing weights or by placing a piece of blotting paper under the knife edge.

2. In reading through the microscope and telescope, exact corresponding positions on scale or position with respect to selected points should be carefully observed.

3. The point of application of stress should be maintained at the center of the bar between the knife edges or if outside of the knife edges, equally distant from same.

4. Any ends projecting beyond the knife edge should be about the same in length. In Methods I and II the ends projecting should be short.

5. Should the loads used be too heavy, there is a danger of the knife edges sinking into the material, producing lag in restitution of rod, especially in the first reading of the restitution.

6. The bar should be perfectly straight, without a distortion, and uniform in every dimension.

For Method II.

7. The mirrors should be placed near the knife edges. If placed outside a short distance the error is very small.

8. Since the method is very sensitive, the apparatus must be absolutely free from any disturbances.

9. The points of reflection on the mirrors, which are slightly turned, must be approximately determined in order to measure the distance between these points on the mirrors and the distance between the mirror, near the telescope and the scale.

For Method IV.

10. The plane of the mirror must be exactly over the knife edge and perpendicular to the rod.

11. The plane of the scale must be placed over the knife edge and perpendicular to the rod.

VII CONCLUSIONS

According to this investigation these standard methods for the determination of Young's Modulus reveal an uniformity in results within themselves. The averages, however, for the various methods differ slightly from each other even though the data was taken simultaneously. This makes it difficult to choose between them. These differences may be attributed either to constant experimental errors or, what is more likely, to the nature of the method of attack.

In Methods I and II, the bending being non-uniform, there is a changing condition of vertical shearing stresses which not only varies constantly with the different amount of stress applied but also with the thickness of the rod. These varying stresses are distinctly noticeable when there is considerable of the ends of the rod projecting beyond the knife-edges. When the stress is applied in the middle the ends projecting over the knife-edge are subjected to a condition which tends to prevent the rod from bending. To make a mathematical determination of all the component forces in such bending would involve very complex equations of a nature beyond the

scope of this investigation and hence we have only a partial mathematical deduction which, however, leads to good approximate results.

In Method I where the microscope is used to measure the deflection, the results are comparatively uniform. Having a firm support and an instrument of high power, the displacement due to the stress can be read with satisfactory accuracy. The student, realizing the simplicity of the apparatus and being able to observe directly what is taking place, will understand more readily the significance of the displacement on change of stress. The one important caution, in adjusting for a reading, is in making a definite estimation of the relation of the mark selected on the bar to the hair in the microscope. This objection can be greatly remedied by making a few trial observations.

In the method using the two mirrors there are two possible advantages: (1) it is sensitive to the very slightest deformation of the beam, affording the use of very small amounts of stress and (2) in the use of small loads it not only prevents the knife-edge from sinking into the sample but also overcomes the danger of passing the elastic limit of the material. There are a few cautions to be observed. The mirrors must be placed over the knife edge with but little of the bar projecting outside of the supports. If there is any disturbance , as walking about the floor, slamming of doors of the building, or heavy traffic on streets near by, it will set the bar to vibration, preventing any definiteness of reading of the scale through the telescope.

In Methods III and IV applying the stress at the ends of the bar the bending is uniform, thus eliminating the objections as in the case of non-uniform bending as stated above. All the force

acting as constant component along all points of the bar, one is able, mathematically, to account easily and accurately for the amounts and the directions of the forces involved. There are no approximations or any small components neglected, consequently deductions made in such a manner are more accurate mathematically.

In case of finding the displacement by means of the microscope, experimentally this method has the same merits as discussed in Method I. Being of uniform bending with simple apparatus and affording closer contact in the observation and reasoning of the pupil, this method proves most satisfactory.

The last method investigated has not proved so favorable in results and manipulation. Although the results in case of the brass show only a small difference from the average while in case of the steel the variation is considerably greater, there are well grounded objections which tend to rank it low as a good method for laboratory use. The plane of the mirror must be exactly over the knife-edge with its normal parallel to the bar. If the bar bends much due to its own weight, placing the plane of the mirror and locating the normal can be only approximate, which leads to serious results. Even if the bar is perfectly straight it is a matter of a number of trials in order to get good results. The displacement of the bar is a factor in the denominator of the formula with a large numerator, causing if the readings are not accurate, a marked variation in the results.

The results of this investigation indicate that Method III ranks best for most purposes. One must conclude in its favor because not only of its complete mathematical deduction but also of its simplicity in experimental manipulation. Method I is as good

except in that there is more or less of unused portions of ends projecting beyond the knife-edges, preventing uniform bending. Method II, being very sensitive, may be highly recommended to any one who wishes to make extensive investigation in elasticity, especially with small amount of stress. Method IV for any purpose proved the least favorable because it is not an easy matter to adjust accurately the parts for the determination of the deformation of the rod.

In conclusion I extend gratefully the most sincere thanks to Professor A. P. Carman for his interest in this work and for placing at my disposal materials and apparatus necessary for its success. I am no less appreciative to Dr. E. H. Williams for the generosity and enduring tact in his assistance to me to complete the work of the problem.